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CLASS X — MATHEMATICS

Chapter 1: Real Numbers

Complete Study Notes with Solved Examples, Exercise-wise Concept Summary,
Additional Practice Questions & Solutions

What this chapter covers

- Euclid's Division Algorithm (recap) and its use in finding HCF
- The Fundamental Theorem of Arithmetic — prime factorisation
- Using prime factorisation to find HCF and LCM of two or more numbers
- The relationship HCF × LCM = Product of two numbers
- Proving irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and numbers built from them

1. The Fundamental Theorem of Arithmetic

1.1 Core Idea

Every composite number can be broken down (factorised) into a product of prime numbers, and this way of breaking it down is unique — the only thing that can differ is the order in which the primes are written.

This is proved using a 'factor tree': keep splitting a number into two factors until every branch ends in a prime.

Theorem 1.1 (Fundamental Theorem of Arithmetic): Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

General form: if x is composite, we write

$$x = p_1 \times p_2 \times \dots \times p_n \quad \text{where } p_1 \le p_2 \le \dots \le p_n \text{ are primes.}$$

Grouping repeated primes together gives the number as a product of prime powers, e.g. $32760 = 2^3 \times 3^2 \times 5 \times 7 \times 13$.

1.2 Worked Examples from the NCERT Text

Example 1: Can 4^n ever end in the digit 0?

A number ends in 0 only if it is divisible by both 2 and 5 (i.e. by 10).

$4^n = (2)^{2n}$, so its only prime factor is 2 — the prime 5 never appears.

By the uniqueness part of the Fundamental Theorem of Arithmetic, no other prime can sneak into the factorisation. Hence 4^n can never end in 0, for any natural number n.

Example 2: LCM and HCF of 6 and 20 (prime factorisation method)

$6 = 2^1 \times 3^1 \qquad 20 = 2^2 \times 5^1$

HCF = product of the smallest power of each COMMON prime = $2^1 = 2$

LCM = product of the greatest power of each prime present = $2^2 \times 3^1 \times 5^1 = 60$

Example 3: HCF and LCM of 96 and 404

$96 = 2^5 \times 3 \qquad 404 = 2^2 \times 101$

HCF(96, 404) = $2^2 = 4$

LCM(96, 404) = $(96 \times 404) \div \text{HCF} = (96 \times 404) \div 4 = 9696$

Example 4: HCF and LCM of 6, 72 and 120

$6 = 2 \times 3 \qquad 72 = 2^3 \times 3^2 \qquad 120 = 2^3 \times 3 \times 5$

HCF = $2^1 \times 3^1 = 6$ (smallest powers of common primes 2 and 3)

LCM = $2^3 \times 3^2 \times 5^1 = 360$ (greatest powers of all primes present)

Important Remark: For three numbers, the product of the numbers is NOT equal to the product of their HCF and LCM. That identity works only for pairs of numbers.

1.3 Key Formulas

For any two positive integers a and b: HCF(a, b) × LCM(a, b) = a × b

HCF = product of the smallest power of each common prime factor.

LCM = product of the greatest power of each prime factor that appears in any of the numbers.

Caution: the HCF × LCM = product rule does NOT extend directly to three or more numbers (see the 'Note to the Reader' box at the end of this document for the correct three-number formulas).

1.4 Exercise 1.1 — Concept Notes

This exercise tests direct application of the Fundamental Theorem of Arithmetic:

- Writing numbers as a product of prime powers (factor tree or repeated division method).
- Finding HCF and LCM using the prime-factorisation method, and verifying HCF × LCM = product of the two numbers.
- Finding HCF/LCM of three numbers using smallest/greatest common prime powers.
- Using HCF(a,b) × LCM(a,b) = a×b to find one when the other is known — no need to factorise!
- Using the 'ends in 0 needs both 2 and 5' logic to test divisibility patterns.
- Recognising that a number of the form (product of several numbers) + (small number) is composite if a common factor can be pulled out of both parts.
- Real-life LCM application: 'meeting again' / 'ringing together' problems — the answer is always the LCM of the individual times/periods.

Additional Practice Questions — Exercise 1.1

Q. 1. Express 5005 as a product of its prime factors.

Solution: 5005 = 5 × 1001 = 5 × 7 × 143 = 5 × 7 × 11 × 13

Q. 2. Find the HCF and LCM of 24 and 36 using prime factorisation, and verify HCF × LCM = 24 × 36.

$24 = 2^3 \times 3$ $36 = 2^2 \times 3^2$

HCF = $2^2 \times 3 = 12$ LCM = $2^3 \times 3^2 = 72$

Check: HCF × LCM = 12 × 72 = 864 = 24 × 36 ✓

Q. 3. The HCF of two numbers is 18 and their LCM is 720. If one of the numbers is 72, find the other.

Using HCF × LCM = product of numbers:

18 × 720 = 72 × (other number)

Other number = (18 × 720) ÷ 72 = 12960 ÷ 72 = 180

Q. 4. Three bells toll at intervals of 9, 12 and 15 minutes respectively. If they toll together at 8:00 a.m., after how many minutes will they toll together again?

$9 = 3^2$, $12 = 2^2 \times 3$, $15 = 3 \times 5$

LCM = $2^2 \times 3^2 \times 5 = 180$

The bells will toll together again after 180 minutes, i.e. at 11:00 a.m.

Q. 5. Show that 12^n cannot end with the digit 0 for any natural number n.

$12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ — its only prime factors are 2 and 3.

Since the prime 5 never occurs in the factorisation, by the uniqueness part of the Fundamental Theorem of Arithmetic, 12^n is never divisible by 10, and so can never end in 0.

Q. 6. Find the LCM and HCF of 40, 36 and 126.

$40 = 2^3 \times 5$ $36 = 2^2 \times 3^2$ $126 = 2 \times 3^2 \times 7$

HCF = $2^1 = 2$ (only 2 is common to all three; lowest power)

LCM = $2^3 \times 3^2 \times 5 \times 7 = 2520$

2. Revisiting Irrational Numbers

2.1 Core Idea

A number 's' is irrational if it CANNOT be written in the form p/q where p and q are integers and $q \neq 0$. Examples: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{15}$, π, $-\sqrt{2}/\sqrt{3}$, and non-terminating non-repeating decimals like 0.10110111011110…

Every proof of irrationality in this chapter uses 'proof by contradiction': assume the number IS rational, show this leads to a logical contradiction, and conclude the assumption was false.

2.2 Theorem 1.2 — The key lemma

Theorem 1.2: Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

This is the tool that makes every irrationality proof work — it lets us 'pull the square root's prime factor out' of a squared term.

2.3 Theorem 1.3 — $\sqrt{2}$ is irrational (proof outline)

- Assume, to the contrary, $\sqrt{2} = r/s$ in lowest terms (r, s coprime).
- Then $r = \sqrt{2}\,s$, so $r^2 = 2s^2 \rightarrow$ 2 divides $r^2 \rightarrow$ (by Theorem 1.2) 2 divides r. Write $r = 2c$.
- Substitute: $4c^2 = 2s^2 \rightarrow s^2 = 2c^2 \rightarrow$ 2 divides $s^2 \rightarrow$ 2 divides s.
- So both r and s are divisible by 2 — contradicting that r, s were coprime.
- Hence the assumption is false, and $\sqrt{2}$ is irrational.

The identical method proves $\sqrt{3}$, $\sqrt{5}$, and in general $\sqrt{p}$ is irrational for any prime p (Example 5 in the text proves $\sqrt{3}$ this way).

2.4 Extending to Combinations (Examples 6 & 7)

Example 6: Show that $5 - \sqrt{3}$ is irrational

Assume $5 - \sqrt{3} = a/b$ (rational). Rearranging: $\sqrt{3} = 5 - a/b$, a rational number (since a, b, 5 are all rational).

This contradicts the already-proven fact that $\sqrt{3}$ is irrational. So $5 - \sqrt{3}$ must be irrational.

Example 7: Show that $3\sqrt{2}$ is irrational

Assume $3\sqrt{2} = a/b$ (rational). Rearranging: $\sqrt{2} = a/(3b)$, which is rational.

This contradicts $\sqrt{2}$ being irrational. So $3\sqrt{2}$ is irrational.

2.5 Two Useful General Facts (state and use — don't need to reprove each time)

- The sum or difference of a rational number and an irrational number is always irrational.
- The product or quotient of a non-zero rational number and an irrational number is always irrational.

These let you instantly justify statements like '$7 + \sqrt{5}$ is irrational' or '$(2/3)\sqrt{7}$ is irrational' without repeating the full contradiction proof, though CBSE typically still expects the proof to be written out in full for exam answers.

2.6 Exercise 1.2 — Concept Notes

This exercise is entirely about writing rigorous 'proof by contradiction' arguments. A complete answer always has this shape:

1. Assume the given number is rational, i.e. equal to a/b with a, b coprime integers, $b \neq 0$.

2. Rearrange the equation to isolate a known irrational (usually $\sqrt{2}$, $\sqrt{3}$ or $\sqrt{5}$) on one side.
3. Argue that the other side is rational (ratio/combination of integers), so the isolated irrational would have to be rational too.
4. This contradicts the known irrationality of $\sqrt{2}$ / $\sqrt{3}$ / $\sqrt{5}$ — so the original assumption is false.
5. Conclude: the given number is irrational.

Common mistake to avoid: never write '$\sqrt{2}$ is irrational because it never ends' as your only justification — CBSE wants the formal coprime-contradiction argument, using Theorem 1.2 where a square root is proved from scratch, or reducing to an already-proved irrational otherwise.

Additional Practice Questions — Exercise 1.2

Q. 1. Prove that $\sqrt{7}$ is irrational.

Assume $\sqrt{7} = a/b$, a, b coprime integers, $b \neq 0$.

Then $a = \sqrt{7}\, b \rightarrow a^2 = 7b^2 \rightarrow 7$ divides $a^2 \rightarrow 7$ divides a (Theorem 1.2, $p = 7$). Write $a = 7c$.

Substituting: $49c^2 = 7b^2 \rightarrow b^2 = 7c^2 \rightarrow 7$ divides $b^2 \rightarrow 7$ divides b.

So a and b share the common factor 7 — contradicting that they are coprime.

Hence $\sqrt{7}$ is irrational.

Q. 2. Prove that $2 + 3\sqrt{5}$ is irrational.

Assume $2 + 3\sqrt{5} = a/b$ (rational). Rearranging: $\sqrt{5} = (a - 2b) / 3b$, a ratio of integers, i.e. rational.

This contradicts the fact that $\sqrt{5}$ is irrational (proved in Exercise 1.2, Q1 of the textbook).

Hence $2 + 3\sqrt{5}$ is irrational.

Q. 3. Prove that $4 - 5\sqrt{2}$ is irrational.

Assume $4 - 5\sqrt{2} = a/b$ (rational). Rearranging: $\sqrt{2} = (4b - a) / 5b$, which is rational.

This contradicts $\sqrt{2}$ being irrational. Hence $4 - 5\sqrt{2}$ is irrational.

Q. 4. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Assume $\sqrt{2} + \sqrt{3} = a/b$ (rational). Squaring both sides:

$2 + 3 + 2\sqrt{6} = a^2/b^2 \rightarrow 5 + 2\sqrt{6} = a^2/b^2 \rightarrow \sqrt{6} = (a^2/b^2 - 5) / 2$, which is rational.

But $\sqrt{6} = \sqrt{2} \times \sqrt{3}$ is irrational (product of the square root of a non-perfect-square number is irrational — can itself be shown by contradiction similarly to $\sqrt{2}$).

This is a contradiction, so $\sqrt{2} + \sqrt{3}$ is irrational.

Q. 5. Show that $1/\sqrt{5}$ is irrational.

Assume $1/\sqrt{5} = a/b$ (rational, coprime a, b). Then $\sqrt{5} = b/a$, a ratio of integers, i.e. rational.

This contradicts $\sqrt{5}$ being irrational. Hence $1/\sqrt{5}$ is irrational.

3. Chapter Summary & Quick-Reference Sheet

3.1 Formula Sheet

Concept	Formula / Statement
HCF (prime factorisation)	Product of smallest power of each COMMON prime factor
LCM (prime factorisation)	Product of greatest power of each prime factor appearing in any number
Relation (2 numbers only)	HCF(a,b) × LCM(a,b) = a × b
Ends in digit 0 test	A number ends in 0 only if BOTH 2 and 5 are its prime factors
Irrational + / − Rational	Always irrational
Irrational × / ÷ nonzero Rational	Always irrational
Theorem 1.2	p prime, $p \mid a^2 \Rightarrow p \mid a$

3.2 Three-Number HCF/LCM Formulas (Note to the Reader)

HCF(p, q, r) × LCM(p, q, r) ≠ p × q × r in general — this shortcut only works for exactly two numbers. For three numbers, use:

LCM(p,q,r) = [p·q·r · HCF(p,q,r)] ÷ [HCF(p,q) · HCF(q,r) · HCF(p,r)]

HCF(p,q,r) = [p·q·r · LCM(p,q,r)] ÷ [LCM(p,q) · LCM(q,r) · LCM(p,r)]

3.3 Common Exam Pitfalls

- Forgetting that HCF × LCM = product rule applies only to TWO numbers, not three or more.
- Writing 'p is prime' as an assumption without stating coprimality of a and b at the start of contradiction proofs.
- Skipping the step that invokes Theorem 1.2 explicitly when proving a fresh √p is irrational — examiners look for this citation.
- Confusing 'composite number' proofs (Q6 type, e.g. 7×11×13+13) — remember: if every term in a sum shares a common factor greater than 1, the whole sum is composite.
- Not reducing r/s or a/b to lowest terms (coprime) before starting the contradiction argument.

3.4 One More Solved 'Composite Number' Style Question

Q. Explain why 5 × 7 × 11 + 7 is a composite number.

5 × 7 × 11 + 7 = 7 × (5 × 11 + 1) = 7 × 56 = 392

Since the expression can be written as 7 × 56, it has factors other than 1 and itself — hence it is composite.

— End of Chapter 1 Notes —