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CLASS XI — MATHEMATICS

Chapter 1: Sets

Complete Study Notes with Solved Examples, Exercise-wise Concept Summary,

Additional Practice Questions & Solutions

What this chapter covers

- Meaning of a set as a 'well-defined collection of objects', and the two ways to represent a set (roster & set-builder)
- Special number sets N, Z, Q, R, Z+, Q+, R+ and the empty, finite & infinite sets
- Equal sets, subsets, proper subsets, singleton sets, and intervals as subsets of R
- Universal set and Venn diagrams
- Operations on sets — union, intersection, difference and complement
- Important laws: commutative, associative, distributive, De Morgan's laws

1. Sets and their Representations

1.1 Core Idea

A set is a well-defined collection of objects — meaning that, given any object, we can always decide for certain whether it belongs to the collection or not. 'The five most talented writers' is NOT a set (opinion-based), but 'the vowels in the English alphabet' IS a set.

Definition: A SET is a well-defined collection of objects. Objects, elements, and members of a set are synonymous.

Notation conventions: Sets are named with capital letters (A, B, X...); elements with small letters (a, b, x...). If a is an element of set A we write $a \in A$ ('a belongs to A'); if not, $b \notin A$.

1.2 Standard Number Sets (memorise these symbols)

Symbol	Meaning
N	Set of all natural numbers
Z	Set of all integers
Q	Set of all rational numbers
R	Set of all real numbers
Z+	Set of positive integers
Q+	Set of positive rational numbers
R+	Set of positive real numbers

1.3 Two Ways to Represent a Set

(i) Roster (Tabular) Form

All elements are listed, separated by commas, inside braces { }. Order doesn't matter and elements are never repeated. E.g. the set of letters in 'SCHOOL' = {S, C, H, O, L}.

(ii) Set-Builder Form

Elements are described by a common property instead of being listed. E.g. V = {x : x is a vowel in the English alphabet}. The colon ':' is read 'such that'.

1.4 Worked Examples from the NCERT Text

Example 1: Roster form of the solution set of $x^2 + x - 2 = 0$

$(x - 1)(x + 2) = 0 \rightarrow x = 1, -2$. Roster form: $\{1, -2\}$

Example 2: Roster form of $\{x : x$ is a positive integer and $x^2 < 40\}$

Required numbers: 1, 2, 3, 4, 5, 6. Roster form: {1, 2, 3, 4, 5, 6}

Example 3: Set-builder form of A = {1, 4, 9, 16, 25, ...}

A = {x : x is the square of a natural number}, or $A = \{x : x = n^2, n \in N\}$

Example 4: Set-builder form of {1/2, 2/3, 3/4, 4/5, 5/6, 6/7}

Each term's numerator is one less than its denominator, running from 1 to 6:

$\{x : x = n/(n+1)$, where n is a natural number and $1 \le n \le 6\}$

Example 5: Matching roster form with set-builder form

{P,R,I,N,C,A,L} matches 'letters of PRINCIPAL' (9 letters, P and I repeat).

{0} matches 'x is an integer and $x + 1 = 1$' (gives $x = 0$).

{1,2,3,6,9,18} matches 'divisors of 18'. {3,−3} matches '$x^2 - 9 = 0$'.

1.5 Exercise 1.1 — Concept Notes

- Deciding 'is it a set?' always comes down to: is the collection WELL-DEFINED (unambiguous membership test)? Subjective criteria like 'most talented' or 'most dangerous' fail this test.
- $\in$ / $\notin$ questions are simple membership checks against a listed set.
- Converting set-builder → roster: work out exactly which values satisfy the condition, then list them.
- Converting roster → set-builder: find the single common property shared by every listed element and no other object.
- Matching questions: check each roster set against each property one at a time — elimination is often fastest.

Additional Practice Questions — Exercise 1.1

Q. 1. Write the set $\{x : x$ is a natural number and $x^2 < 30\}$ in roster form.

Natural numbers whose square is less than 30: 1, 2, 3, 4, 5 (since $5^2 = 25 < 30$, $6^2 = 36$ not < 30).

Roster form: {1, 2, 3, 4, 5}

Q. 2. Write the set {5, 10, 15, 20, ...} in set-builder form.

{x : x is a positive multiple of 5}, or $\{x : x = 5n, n \in N\}$

Q. 3. Is 'the collection of all tall students in your school' a set? Justify.

No — 'tall' is not well-defined (there's no fixed, universally agreed height cut-off), so membership cannot be decided with certainty for a borderline student.

Q. 4. Write the set of all letters in the word 'ENGINEERING' in roster form.

Distinct letters (repetition removed): {E, N, G, I, R}

2. Empty, Finite, Infinite and Equal Sets

2.1 The Empty Set

Definition 1: A set which does not contain any element is called the empty set (or null set / void set), denoted by ϕ or { }.

Example: {x : x is a natural number, $1 < x < 2$} = ϕ, since no natural number lies strictly between 1 and 2.

2.2 Finite and Infinite Sets

Definition 2: A set which is empty or has a definite (countable) number of elements is finite; otherwise it is infinite.

n(S) denotes the number of distinct elements in set S. If n(S) is a natural number (or 0), S is finite.

Note: NOT every infinite set can be written in roster form with '...' — e.g. the set of real numbers has no describable pattern to list, even though other infinite sets like {1, 3, 5, 7, ...} can be shown this way.

Example 6: Finite or infinite?

$\{x \in N : (x-1)(x-2) = 0\} = \{1, 2\}$ → finite

$\{x \in N : x$ is prime$\}$ → infinite (infinitely many primes)

$\{x \in N : 2x - 1 = 0\} = \phi$ → finite (empty set is finite)

2.3 Equal Sets

Definition 3: Two sets A and B are equal (A = B) if they have exactly the same elements — every element of A is in B and vice versa.

A set does not change if elements are repeated: {1,2,3} = {2,2,1,3,3}.

Example 7: Finding equal sets among A={0}, B={x:x>15,x<5}, C={x:x−5=0}, D={x:x^2=25}, E={x: integral positive root of $x^2-2x-15=0$}

B = ϕ (no number is both >15 and <5). C = {5}. D = {−5, 5}. E = {5}.

Only C and E have exactly the same elements → C = E is the only equal pair.

2.4 Exercise 1.2 — Concept Notes

- Null-set check: does the condition describe a genuinely impossible requirement, or just a set with elements you need to work out?
- Finite vs infinite: ask 'can I, in principle, finish counting the elements?' — geometric objects like 'all lines parallel to an axis' or 'all circles through a point' are infinite.
- Equal-sets check: convert every set to roster form first, then compare element-by-element — order and repetition never matter.

Additional Practice Questions — Exercise 1.2

Q. 1. Is {x : x is a natural number, $x^2 = 3$} a null set?

√3 is not a natural number, so no natural number satisfies $x^2 = 3$. Yes, this is the null set (ϕ).

Q. 2. State whether {x : x is an even prime number} is finite or infinite.

2 is the only even prime number, so the set is {2} — finite (in fact, a singleton set).

Q. 3. Are A = {x : x is a letter in 'ASSASSINATION'} and B = {x : x is a letter in 'STATION'} equal?

A (distinct letters) = {A, S, I, N, T, O}. B (distinct letters) = {S, T, A, I, O, N}.

Both sets contain exactly {A, I, N, O, S, T} → A = B (equal sets).

3. Subsets

3.1 Core Idea

Definition 4: A is a subset of B (written $A \subset B$) if every element of A is also an element of B, i.e. $a \in A \Rightarrow a \in B$.

Every set is a subset of itself ($A \subset A$). The empty set is a subset of every set ($\phi \subset A$).

If $A \subset B$ and $B \subset A$, then A = B. If $A \subset B$ but $A \neq B$, then A is called a proper subset of B, and B is the superset of A.

A singleton set has exactly one element, e.g. {a}.

3.2 Worked Examples

Example 9: Insert $\subset$ or $\not\subset$

$\phi \subset B$ (empty set is subset of every set). A = {1,3}, B = {1,5,9}: $A \not\subset B$ since $3 \notin B$.

$A \subset C$ where C = {1,3,5,7,9} since both 1, 3 $\in$ C. $B \subset C$ since 1, 5, 9 all $\in$ C.

Example 11: If $A \in B$ and $B \subset C$, is $A \subset C$ necessarily true?

No. Counter-example: A = {1}, B = {{1}, 2}, C = {{1}, 2, 3}. Here $A \in B$ and $B \subset C$, but $A \not\subset C$ because $1 \in A$ while $1 \notin C$ (only the SET {1} is an element of C, not the number 1 itself).

3.3 Subsets of R and Intervals

Key chain of subset relations: $N \subset Z \subset Q \subset R$, and the irrationals $T \subset R$, with $N \not\subset T$ (no natural number is irrational).

Interval Notation	Set-builder Form	Description
(a, b)	$\{x : a < x < b\}$	Open interval — excludes both endpoints
[a, b]	$\{x : a \leq x \leq b\}$	Closed interval — includes both endpoints
[a, b)	$\{x : a \leq x < b\}$	Includes a, excludes b
(a, b]	$\{x : a < x \leq b\}$	Excludes a, includes b

The length of any of these intervals is (b − a).

3.4 Exercise 1.3 — Concept Notes

- To decide $A \subset B$: check EVERY element of A against B; one missing element disproves the relation ($A \not\subset B$).
- Careful with { } vs bare elements: {3,4} is a SET and can be a subset; if {3,4} itself appears as a listed element of another set (e.g. A = {1, 2, {3,4}, 5}), then $\{3,4\} \in A$ (element), while $\{\{3,4\}\} \subset A$ (as a set containing that element).
- ϕ is always both an element check ($\phi \in A$ only if A explicitly contains ϕ as a member) and always true as $\phi \subset A$.
- A set with n elements has exactly 2^n subsets (including ϕ and the set itself) — useful for 'write all subsets' questions.
- Interval ↔ set-builder conversion is direct: match ≤ / < with closed/open ends respectively.

Additional Practice Questions — Exercise 1.3

Q. 1. Write all the subsets of {p, q, r}.

Number of subsets = $2^3 = 8$.

ϕ, {p}, {q}, {r}, {p,q}, {q,r}, {p,r}, {p,q,r}

Q. 2. Let A = {1, {2, 3}, 4}. State whether each is true or false: (a) {2,3} $\in$ A (b) {2,3} $\subset$ A (c) 2 $\in$ A

(a) True — {2,3} is listed as one of the three elements of A.

(b) False — for {2,3} $\subset$ A we would need both 2 $\in$ A and 3 $\in$ A individually, but they are not (only the set {2,3} is an element).

(c) False — 2 itself is not a listed element of A; only the set {2,3} is.

Q. 3. Write {x : x $\in$ R, $-8 \le x < 15$} as an interval.

Closed at −8 (≤), open at 15 (<) → interval form: $[-8, 15)$

Q. 4. Write the interval $(-\infty, 4]$ in set-builder form.

{x : x $\in$ R, $x \le 4$}

4. Venn Diagrams and Operations on Sets

4.1 Universal Set and Venn Diagrams

The Universal Set (U) is the 'basic' set relevant to a particular discussion, containing all objects under consideration; every other set in that context is a subset of U. It is usually drawn as a rectangle in a Venn diagram, with subsets shown as circles inside it.

4.2 Union of Sets (∪)

Definition 5: $A \cup B = \{x : x \in A \text{ or } x \in B\}$ — all elements in A, in B, or in both (common elements counted once).

Properties of Union:

- $A \cup B = B \cup A$ (Commutative law)
- $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
- $A \cup \phi = A$ (Identity law — ϕ is the identity element for ∪)
- $A \cup A = A$ (Idempotent law)
- $U \cup A = U$ (Law of U)

Key fact: if $B \subset A$, then $A \cup B = A$ (shown in Example 13 of the text using vowels).

4.3 Intersection of Sets (∩)

Definition 6: $A \cap B = \{x : x \in A \text{ and } x \in B\}$ — only the elements common to both A and B.

Properties of Intersection:

- $A \cap B = B \cap A$ (Commutative law)
- $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)
- $\phi \cap A = \phi$, $U \cap A = A$ (Laws of ϕ and U)
- $A \cap A = A$ (Idempotent law)
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law — ∩ distributes over ∪)

If $A \cap B = \phi$, then A and B are called disjoint sets — they share no common element.

Key fact: if $B \subset A$, then $A \cap B = B$ (shown in Example 17 of the text).

4.4 Difference of Sets

$A - B = \{x : x \in A \text{ and } x \notin B\}$ — elements in A but NOT in B. Note: $A - B \neq B - A$ in general.

The three sets $A - B$, $A \cap B$, and $B - A$ are mutually disjoint — they never overlap with each other, and together they make up $A \cup B$.

4.5 Worked Examples

Example 14: X = {Ram, Geeta, Akbar} (hockey), Y = {Geeta, David, Ashok} (football). Find $X \cup Y$.

$X \cup Y$ = {Ram, Geeta, Akbar, David, Ashok} — students in hockey OR football OR both.

Example 16: Find $X \cap Y$ for the same sets.

$X \cap Y$ = {Geeta} — the only student in both teams.

Example 18: A = {1,2,3,4,5,6}, B = {2,4,6,8}. Find A − B and B − A.

A − B = {1, 3, 5}   B − A = {8}   (confirms A − B ≠ B − A)

4.6 Exercise 1.4 — Concept Notes

- Union = 'combine and de-duplicate'; Intersection = 'keep only what's shared'; Difference = 'remove B's elements from A'.
- For number-pattern sets (e.g. multiples, primes, naturals in a range), first convert to roster form, THEN apply the operation.
- Disjoint-set checks: compute $A \cap B$; if it's ϕ, the sets are disjoint.
- Watch direction carefully in difference questions — A − B and B − A are generally different answers.
- R − Q (real numbers minus rationals) = the set of irrational numbers, T.

Additional Practice Questions — Exercise 1.4

Q. 1. If A = {2, 3, 5, 7} and B = {1, 2, 3, 4, 5}, find $A \cup B$, $A \cap B$ and A − B.

$A \cup B$ = {1, 2, 3, 4, 5, 7}

$A \cap B$ = {2, 3, 5}

A − B = {7}

Q. 2. If A = {x : x is a multiple of 4, x ≤ 20} and B = {x : x is a multiple of 6, x ≤ 20}, find $A \cap B$.

A = {4, 8, 12, 16, 20}   B = {6, 12, 18}

$A \cap B$ = {12} (the common multiple of 4 and 6 up to 20, i.e. multiples of LCM(4,6)=12)

Q. 3. Are {x : x is a multiple of 3} and {x : x is a multiple of 7} disjoint?

No — they share common multiples of 21 (e.g. 21, 42, 63, ...), so their intersection is non-empty, meaning they are NOT disjoint.

Q. 4. If U = {1,...,10}, A = {1,2,3,4,5}, B = {4,5,6,7}, find (A − B) and (B − A), and verify they are disjoint from $A \cap B$.

A − B = {1, 2, 3}   B − A = {6, 7} $A \cap B$ = {4, 5}

These three sets share no elements with each other — confirming they are mutually disjoint, and together = $A \cup B$ = {1,2,3,4,5,6,7}.

5. Complement of a Set

5.1 Core Idea

Definition 7: $A' = \{x : x \in U \text{ and } x \notin A\} = U - A$ — all elements of the universal set U that are NOT in A.

Example 20: $U = \{1,\ldots,10\}$, $A = \{1,3,5,7,9\}$. Find A'.

$A' = \{2, 4, 6, 8, 10\}$

5.2 Properties of Complement Sets

Law	Statement
Complement laws	$A \cup A' = U$, $A \cap A' = \phi$
De Morgan's Law 1	$(A \cup B)' = A' \cap B'$
De Morgan's Law 2	$(A \cap B)' = A' \cup B'$
Double complementation	$(A')' = A$
Empty/Universal set laws	$\phi' = U$, $U' = \phi$

De Morgan's Laws in words: the complement of a union is the intersection of the complements; the complement of an intersection is the union of the complements. (Named after mathematician Augustus De Morgan.)

Example 22: Verify $(A \cup B)' = A' \cap B'$ for $U=\{1,\ldots,6\}$, $A=\{2,3\}$, $B=\{3,4,5\}$

$A' = \{1,4,5,6\}$, $B' = \{1,2,6\} \rightarrow A' \cap B' = \{1, 6\}$

$A \cup B = \{2,3,4,5\} \rightarrow (A \cup B)' = \{1, 6\}$

Both equal $\{1, 6\}$ ✓— De Morgan's law verified.

5.3 Exercise 1.5 — Concept Notes

- Complement always needs a stated (or clearly implied) universal set U — 'complement' has no meaning without it.
- Fastest method: $A' = U - A$, so just list what's in U but not in A.
- For 'natural numbers as universal set' style questions, describe the complement in words using the negation of the original property (e.g. complement of 'even' is 'odd').
- Use De Morgan's laws to shortcut $(A \cup B)'$ or $(A \cap B)'$ computations instead of finding the union/intersection first and then complementing.

Additional Practice Questions — Exercise 1.5

Q. 1. $U = \{1,2,\ldots,12\}$, $A = \{2,4,6,8,10,12\}$, $B = \{3,6,9,12\}$. Find A', B', and verify $(A \cap B)' = A' \cup B'$.

$A' = \{1,3,5,7,9,11\}$ $B' = \{1,2,4,5,7,8,10,11\}$

$A' \cup B' = \{1,2,3,4,5,7,8,9,10,11\}$

$A \cap B = \{6, 12\} \rightarrow (A \cap B)' = \{1,2,3,4,5,7,8,9,10,11\}$

Both match ✓— De Morgan's law verified.

Q. 2. Taking N as the universal set, find the complement of $\{x : x \text{ is a perfect square}\}$.

Complement = {x : x ∈ N and x is NOT a perfect square}, i.e. all natural numbers except 1, 4, 9, 16, 25, ...

Q. 3. If U is the set of all quadrilaterals and A is the set of all quadrilaterals that are NOT rectangles, describe A′.

A′ = the set of all quadrilaterals that ARE rectangles (the complement negates the defining condition).

6. Miscellaneous Exercise — Concept Notes

The Miscellaneous Exercise moves from computing with specific sets to PROVING general set-theory statements (subset relations, set equalities) using logical argument — a first taste of mathematical proof.

- Standard proof technique for 'show $A = B$': pick an arbitrary $a \in A$, show $a \in B$ (so $A \subset B$); then pick an arbitrary $b \in B$, show $b \in A$ (so $B \subset A$). Together, $A = B$.
- To disprove a general claim (e.g. 'if $A \not\subset B$ and $B \not\subset C$ then $A \not\subset C$'), it is enough to find ONE counter-example with specific sets.
- Four-way equivalence proofs (like $A \subset B \Leftrightarrow A - B = \phi \Leftrightarrow A \cup B = B \Leftrightarrow A \cap B = A$) are typically shown as a logical cycle: (i)⇒(ii)⇒(iii)⇒(iv)⇒(i).

Worked Example from the Text — Example 25

Show that $A \cup B = A \cap B$ implies $A = B$.

Let $a \in A$. Then $a \in A \cup B = A \cap B$, so $a \in B$. Hence $A \subset B$.

Let $b \in B$. Then $b \in A \cup B = A \cap B$, so $b \in A$. Hence $B \subset A$.

Since $A \subset B$ and $B \subset A$, we conclude $A = B$.

Additional Practice Questions — Miscellaneous Exercise

Q. 1. If $A \subset B$, show that $A \cup B = B$.

$A \cup B$ always contains B. To show $A \cup B \subset B$: let $x \in A \cup B$. Then $x \in A$ or $x \in B$.

If $x \in A$, then since $A \subset B$, $x \in B$. If $x \in B$, trivially $x \in B$. Either way $x \in B$, so $A \cup B \subset B$.

Since $B \subset A \cup B$ and $A \cup B \subset B$, we get $A \cup B = B$.

Q. 2. Give an example of sets A, B, C such that $A \cap B = A \cap C$ but $B \neq C$.

Let $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{2, 4\}$.

$A \cap B = \{2\}$, $A \cap C = \{2\}$ — equal. But $B = \{2,3\} \neq \{2,4\} = C$.

This shows $A \cap B = A \cap C$ does NOT by itself force $B = C$.

Q. 3. Show that $A - B = A \cap B'$.

$A - B = \{x : x \in A \text{ and } x \notin B\} = \{x : x \in A \text{ and } x \in B'\}$ (since $x \notin B$ means $x \in B'$ when $B' = U - B$)

This is exactly the definition of $A \cap B'$. Hence $A - B = A \cap B'$.

6.1 Chapter Summary

- A set is a well-defined collection of objects.
- A set with no elements is the empty set (ϕ); a set with a countable number of elements is finite, otherwise infinite.
- Two sets are equal if they have exactly the same elements.
- $A \subset B$ means every element of A is also in B; intervals are subsets of R.
- $A \cup B$: elements in A or B. $A \cap B$: elements common to both. $A - B$: elements in A but not B.
- $A' = U - A$ is the complement of A with respect to universal set U.
- De Morgan's Laws: $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

6.2 Quick-Reference Formula Sheet

Concept	Formula / Rule
Number of subsets of an n-element set	2^n
Union identity/idempotent	$A \cup \phi = A$, $A \cup A = A$
Intersection identity/idempotent	$U \cap A = A$, $A \cap A = A$, $\phi \cap A = \phi$
Distributive law	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Complement laws	$A \cup A' = U$, $A \cap A' = \phi$, $(A')' = A$
De Morgan's Laws	$(A\cup B)' = A'\cap B'$, $(A\cap B)' = A'\cup B'$
Difference as complement	$A - B = A \cap B'$
Mutually disjoint triple	$A-B$, $A\cap B$, $B-A$ never overlap

6.3 Common Exam Pitfalls

- Confusing 'element of' (∈) with 'subset of' (⊂) — especially when a set contains another set as one of its elements, e.g. A = {1, {2,3}, 4}.
- Forgetting that $\phi \subset$ every set, but $\phi \in A$ only if ϕ is explicitly listed as an element of A.
- Sign errors in $A - B$ vs $B - A$ — always re-read which set comes first.
- Applying De Morgan's laws backwards (mixing up ∪ and ∩ on the two sides).
- Treating an infinite set as impossible to reason about — infinite sets can still have precise subset/superset/equality relationships proved logically, even without listing all elements.

— End of Chapter 1 Notes —